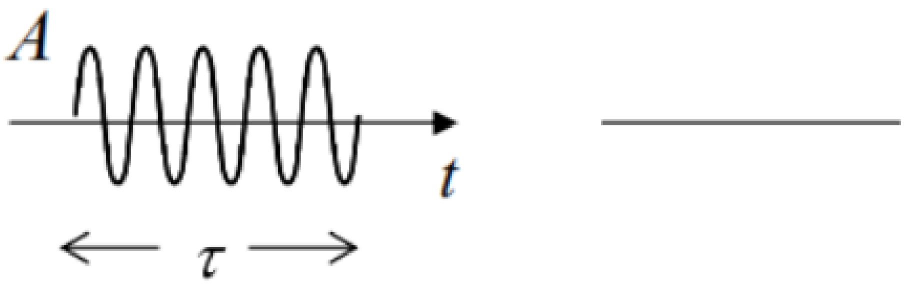
Binary Amplitude Shift Keying (BASK): Signal Representation

Send: $s_1(t) = A\cos(2\pi f_c t)$, if the information bit is "1" $\Rightarrow E_1 = \frac{A^2\tau}{2}$

Send: $s_2(t) = 0$, if the information bit is "0"; $\Rightarrow E_2 = 0$

The average energy per bit $E_b = \frac{1}{2}(E_1 + E_2) = \frac{A^2\tau}{4}$

"1" "0"



$s_1(t) = A\cos(2\pi f_c t)$ $s_2(t) = 0$

$0 \le t \le \tau$

$\tau = nT_c$

τ: is the time allocated to transmit the binary digit.

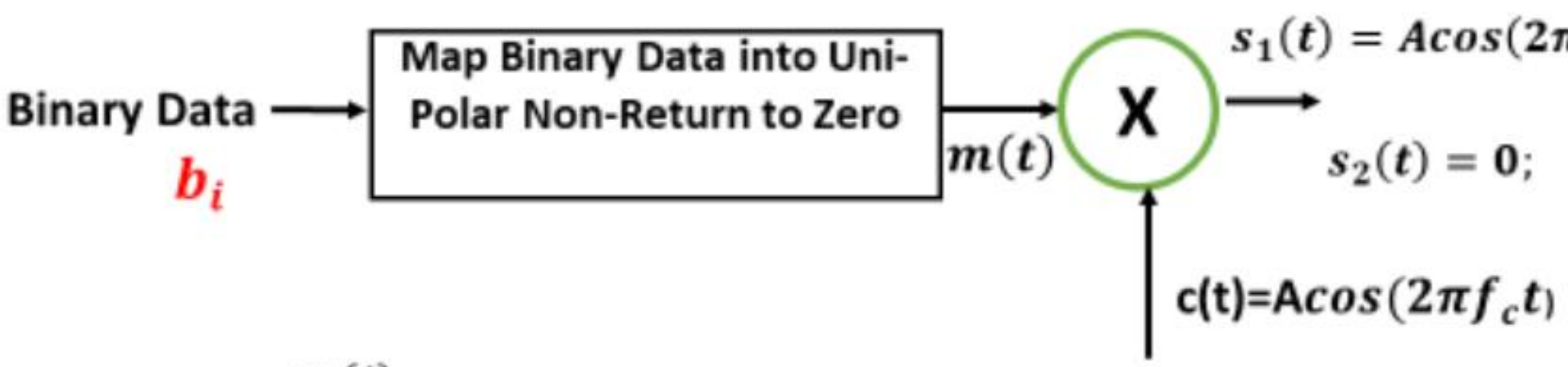
$T_c = 1/f_c$ is the carrier period

$R_b = \frac{1}{\tau}$: *Data rate* bits/sec

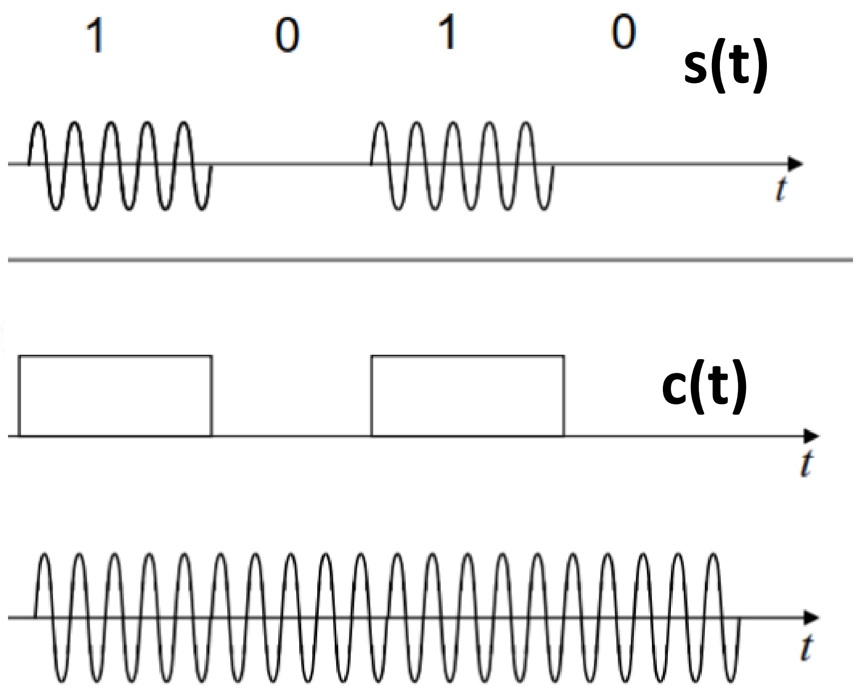
$\tau = nT_c$

In this figure n=5

Binary Amplitude Shift Keying : Generation



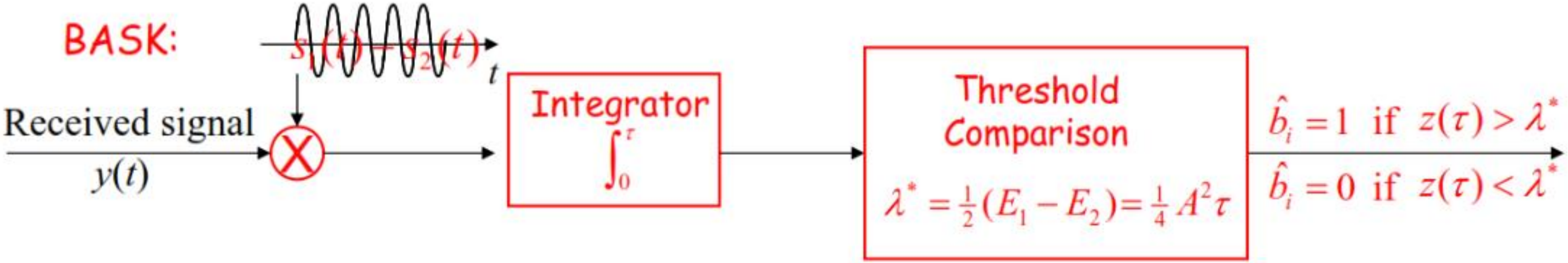
m(t): Unipolar non-return to zero



$$s(t) = m(t)c(t)$$

NOTE: BASK is also called ON-OFF Keying

Binary Amplitude Shift Keying : The Optimum Receiver



Optimum Receiver Implemented as a Correlator Followed by a Threshold Detector

Probability of Error:

$$P_b^* = Q\left(\sqrt{\frac{\int_0^\tau (s_1(t) - s_2(t))^2 dt}{2N_0}}\right)$$

$$E_1 = \int_0^\tau (s_1(t))^2 dt$$

With $\tau = nT_c$

$$E_1 = A^2\tau/2$$

$$s_1(t) = A\cos(2\pi f_c t)$$

$$s_2(t) = 0,$$

$$E_1 = \frac{A^2\tau}{2}$$

$$E_2 = 0$$

$$E_b = \frac{1}{2}(E_1 + E_2) = \frac{A^2\tau}{4}$$

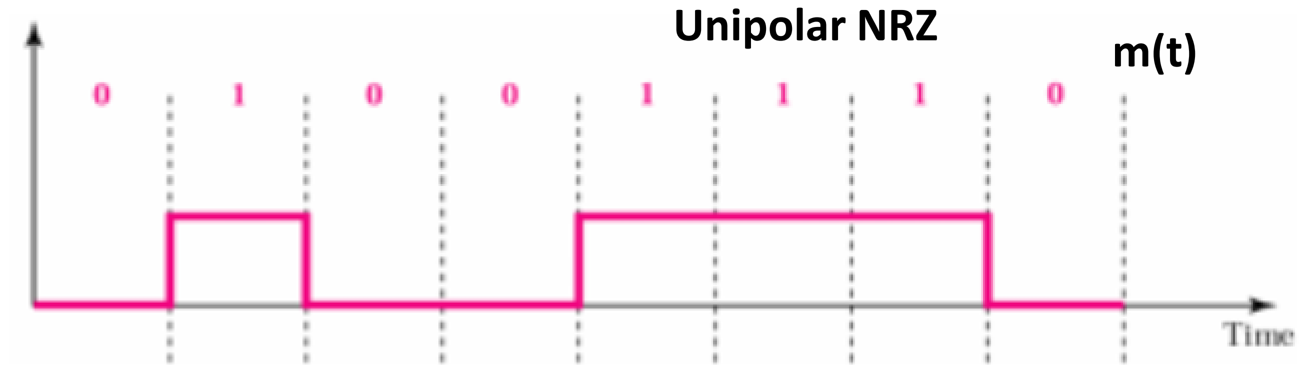
Optimal BER:

$$P_b^* = Q\left(\sqrt{\frac{A^2\tau}{4N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Binary Amplitude Shift Keying: Power Spectral Density

Let $m(t)$ be the unipolar NRZ signal with autocorrelation function $R_m(\tau)$ and power spectral density $G_m(f)$.

- You can easily verify that the unipolar non-return to zero signal $m(t)$ is related to the polar non-return signal $m'(t)$ (used in the generation of the BPSK) by:



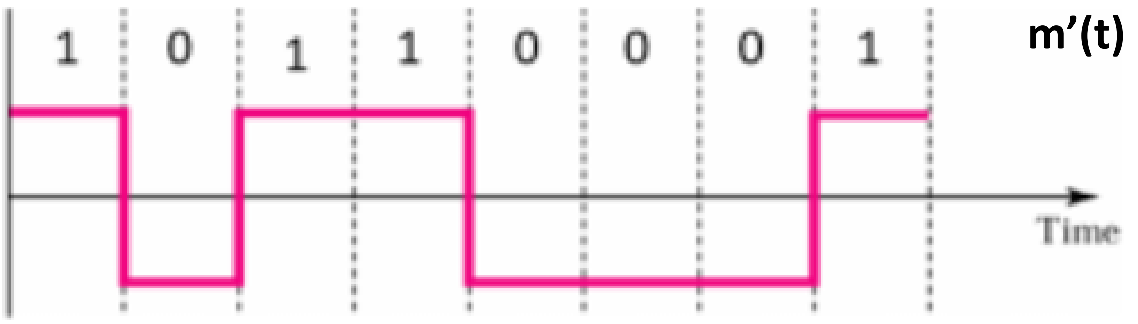
- $m(t) = \frac{1}{2}(1 + m'(t))$

The autocorrelation function of m(t) is

- $R_Y(\tau) = E\{m(t)m(t+\tau)\} =$

$$= E\left\{\frac{1}{2}(1+m'(t))\frac{1}{2}(1+m'(t+\tau))\right\} = \frac{1}{4} + \frac{1}{4}R_{m'}(\tau);$$

Note that for the polar-NRZ $\boldsymbol{E\{m'(t)\} = 0}$



- The power spectral density of m(t) is:

- $G_m(f) = \frac{1}{4}\delta(f) + \frac{1}{4}G_{m'}(f)$

Binary Amplitude Shift Keying: Power Spectral Density

The Wiener –Khintchine Thorem: The power spectral density $G_X\,(f)$ and the autocorrelation function $R_X(\tau)$ of a stationary random process $X(t)$ form a Fourier transform pairs:

- $G_X(f) = \int_{-\infty}^{\infty} R_X\,(\tau)e^{-j2\pi f\tau}d\tau$ (Fourier Transform)
- $R_X(\tau) = \int_{-\infty}^{\infty} G_X\,(f)e^{j2\pi f\tau}df$ (Inverse Fourier Transform)

- The power spectral density of m(t), the unipolar NRZ is:
- $\boldsymbol{G_m(f) = \frac{1}{4}\delta(f) + \frac{1}{4}G_{m'}(f)}$
- In the previous video we saw that if a random process $X(t)$ with an autocorrelation function $R_X(\tau)$ and a power spectral density $G_X(f)$ is mixed with a sinusoidal function $\cos(2\pi f_c t + \theta)$; θ is a r.v uniformly distribution over $(0, 2\pi)$ to form a new process

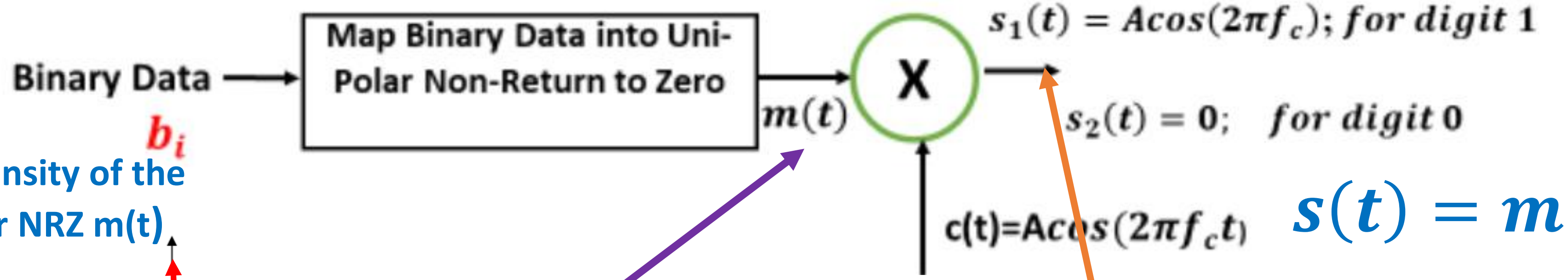
$$Y(t) = X(t)\cos(2\pi f_c t + \theta).$$

Our Problem: $\boldsymbol{s(t) = m(t)\cos(2\pi f_c t + \theta)}$

then the autocorrelation function and power spectral density of Y(t) are given by:

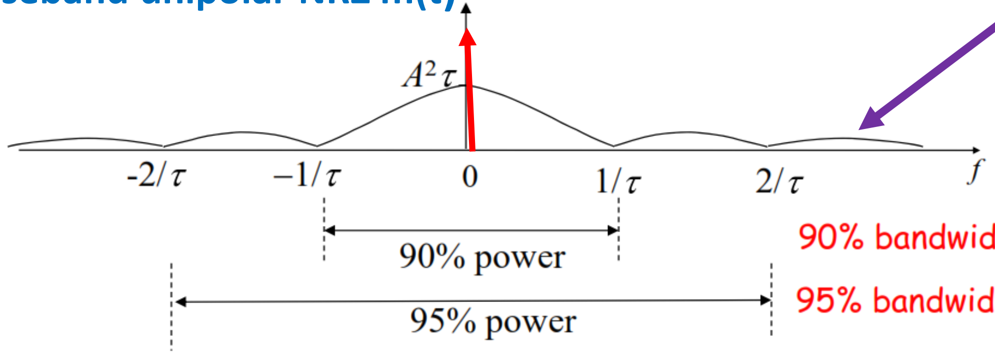
- $R_Y(\tau) = E\{\,Y(t)Y(t+\tau)\} = \frac{R_X(\tau)}{2} \cdot \boldsymbol{cos2\pi f_c\tau}$;
- $\boldsymbol{G_Y(f) = \frac{1}{4}\{G_X(f - f_c) + G_X(f + f_c)\}}$
- **Hence,** $\boldsymbol{G_{BASK}(f) = \frac{1}{4}\{G_m(f - f_c) + G_m(f + f_c)}$

Binary Amplitude Shift Keying : Power Spectral Density and Bandwidth



$$s(t) = m(t)\cos(2\pi f_c t)$$

Power Spectral Density of the baseband unipolar NRZ m(t)



90% bandwidth: $1/\tau$

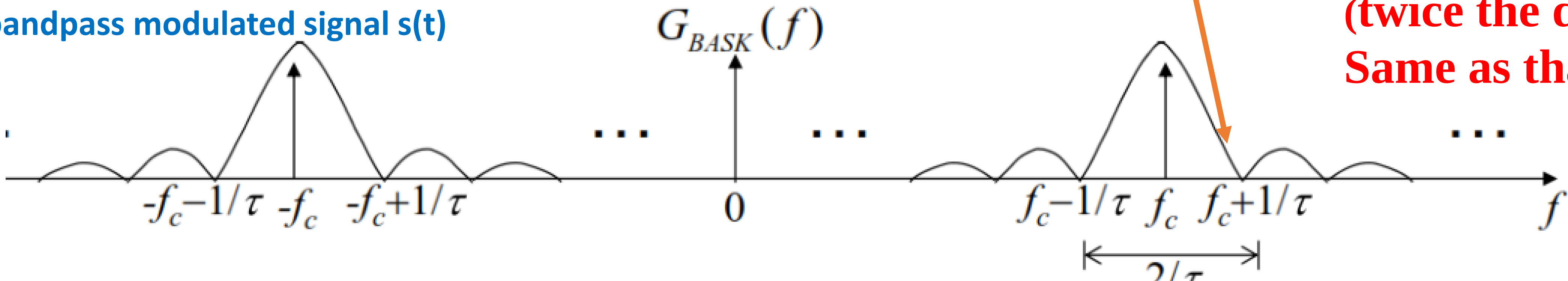
95% bandwidth: $2/\tau$

$$G_{BASK}(f) = \frac{1}{4}[G_{BOOK}(f - f_c) + G_{BOOK}(f + f_c)]$$

Bandwidth of BASK s(t)

(twice the data rate); Same as that of BPSK

Power Spectral Density of the bandpass modulated signal s(t)

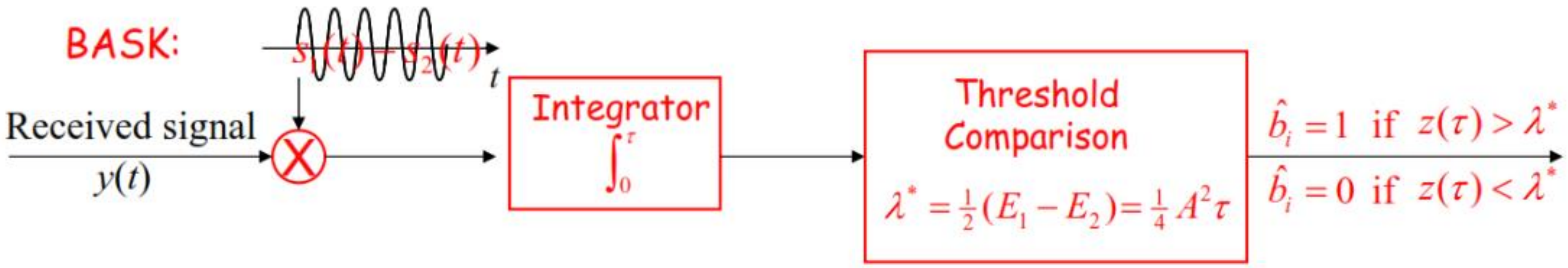


$2R_b$

$$\text{The 90\% power bandwidth} = \frac{2}{\tau} = 2R_b$$

Non-coherent Demodulation of the Binary ASK Signal

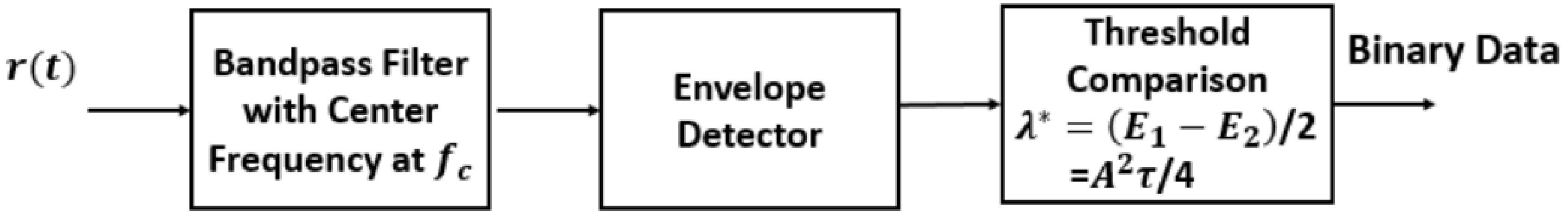
The demodulator which uses the signal difference $s_1(t) - s_2(t) = A cos(2\pi f_c t)$ is called coherent demodulator



In non-coherent demodulation, there is no need for the carrier frequency at the receiver. The basic elements of the receiver are a bandpass filter with center frequency at the carrier, an envelope detector, and a threshold comparator. The receiver is simple, however it is not optimal in terms of the probability of error. The details are shown in the following block diagram

$$\mathbf{r(t)} = \boldsymbol{A cos(2\pi f_c t) + n(t)}$$

$$\mathbf{r(t)} = \boldsymbol{n(t)}$$



Non-Coherent Binary ASK Demodulation